SOLUTIONS PART A – MATHEMATICAL INDUCTION INVESTIGATION

1. Required to prove 12 + 22 +…+n2 =  for all positive integers n..

Step 1 Verify the statement is true when n = 1

L.H.S.= 1

R.H.S. =  = 1

Statement is true for n = 1

Step 2 Assume the statement is true for n = k

That is, 

Step 3 Prove statement true for n = k + 1

That is, prove 

LHS = 

=  from Step 2

=

=

=

=

=

= 

=

=RHS

The statement is true for n = k + 1 if it is true for n = k.

Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence, 12 + 22 +…+n2 =  is true for all positive integers n.

2. Required to prove 13 + 23 +…+n3 =  for all positive integers n.

Step 1 Verify the statement is true when n = 1

L.H.S.= 1

R.H.S. =  = 1

Statement is true for n = 1

Step 2 Assume the statement is true for n = k

That is, 

Step 3 Prove statement true for n = k + 1

That is, prove 

LHS = 

=  from Step 2

=

=

=

=

=

=RHS

The statement is true for n = k + 1 if it is true for n = k.

Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence, 13 + 23 +…+n3 =  is true for all positive integers n.

1. Required to prove  =  for all positive integers n.

Step 1 Verify the statement is true when n = 1

L.H.S.= 2

R.H.S. =  = 2

Statement is true for n = 1

Step 2 Assume the statement is true for n = k

That is,  = 

Step 3 Prove statement true for n = k + 1

That is, prove =

LHS = 

=  +  from Step 2

=

=

=

=RHS

The statement is true for n = k + 1 if it is true for n = k.

Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence, = is true for all positive integers n.

4. Required to prove for all positive integers n.

Step 1 Verify the statement is true when n = 1

L.H.S = 

= 

= 

= 

R.H.S. = 

=

Statement is true for n = 1

Step 2 Assume the statement is true for n = k

That is, 

Step 3 Prove statement true for n = k + 1

That is, prove 

LHS = 

=

=

=

= from Step 2

=

=RHS

The statement is true for n = k + 1 if it is true for n = k.

Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence,  is true for all positive integers n.

5. Required to prove  is divisible by 6 for all positive integers n.

Step 1 Verify the statement is true when n = 1

 = 6

Divisible by 6

Statement is true for n = 1

Step 2 Assume the statement is true for n = k

That is,  is divisible by 6

Step 3 Prove statement true for n = k + 1

That is, prove  is divisible by 6



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=

=

is divisible by 6 from Step 2

 is divisible by 6 if  is even

If is an odd number  is odd so = odd + odd +2 = even

If is an even number  is even so = even + even +2 = even

So  is even for all positive integers  so  is divisible by 6

 is divisible by 6

The statement is true for n = k + 1 if it is true for n = k.

Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence,  is divisible by 6 is true for all positive integers n.

6.



Step 4 As the statement is true for n = 1 it must be true for n =2.

As the statement is true for n = 2 it must be true for n =3 and so on.

Hence  is true for all positive integers n.